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SUBJECT: A Description of Hypersonic Laminar Heat Transfer at the Stagnation Point of a Blunt Body - Case 110

DATE: September 10, 1964

FROM: J. S. Dohnanyi

SUMMARY

24299 ABST

The basic concepts involved in the stagnation point convective heat transfer for a blunt body moving at hypersonic velocity are discussed. Computational procedures are outlined in detail using the heat transfer parameter  $N_u/\sqrt{R_e}$  as the basic quantity of reference. The most recent theories are briefly reviewed; they agree with each other to within  $\pm 25\%$  and are in good agreement with experiment. Since the experimental points scatter over a range of about  $\pm 50\%$  (i. e. a factor of two) and since all the theoretical curves for the heat transfer parameter fall within this range, it is not possible to prefer one theory to any of the others, on the basis of experimental evidence alone. Fay and Kemp use the most recent transport properties in their calculations and obtain conservative results while a not so recent calculation by Hoshizaki (used by NAA for design criteria) predicts heat transfer parameters about 30% lower in the velocity range of 20,000 to 35,000 ft/sec. This writer feels, therefore, that because of existing uncertainties, Hoshizaki's method may underestimate the stagnation point heating rates by as much as 30% at Apollo-type reentry conditions.

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## MEMORANDUM FOR FILE

### INTRODUCTION

As the Apollo Command Module reenters the earth's atmosphere at a velocity of 36,000 ft/sec it will gradually be slowed down by atmospheric drag. The subsequent loss in the kinetic energy of the craft is transferred to the surrounding gases in the form of heat, chemical dissociation, ionization and turbulence. In other words, the gases between the craft and the bow shock will become very hot and most of this heating will develop in the region near the front face of the craft. Because of this condition energy will be transferred back to the craft in the form of radiative and convective heating. In this report we shall outline the procedure to calculate the convective heating at the stagnation point of the craft and discuss the uncertainties involved in the most recent methods available; it turns out that for a stagnation pressure of one atmosphere the best correlation formulas available agree to within about  $\pm 25\%$  with each other.

### FLOW FIELD CONSIDERATIONS

When a blunt body moves through air at hypersonic speed, a bow shock forms ahead of it provided that the ambient air has a density at least as great as that corresponding to an altitude of about 300,000 to 400,000 ft. in the earth's atmosphere.\* Following usual aerodynamic conventions, we assume the test object to be stationary and that the ambient air flows relative to the body.

In figure-1 we give a schematic sketch of the different types of flow for the case of a sphere moving at hypersonic speed. As air moves across the shock front, it is slowed down and the kinetic energy associated with the decrease in the linear velocity is transformed into thermal energy. Most of the hot gases then flow around the body as if the body boundaries exhibited no frictional drag. A small fraction of the gas is, however, subject to viscous damping because of the nonnegligible shearing force exerted by the body wall on the gas. The portion of the flow-field influenced by this frictional force is confined to a thin layer of gas around the body and is known as the boundary layer.

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\*For a discussion of some of the physical aspects of the Apollo-type reentry flow-field, see e. g. references 1 and 2.

The boundary layer may be laminar or turbulent depending on the magnitude of the Reynolds number

$$R_e = \frac{\rho v l}{\mu}$$

where  $\rho$  is the density of the gas,  $v$  is the velocity,  $l$  a characteristic body length and  $\mu$  the coefficient of viscosity. In typical cases, the flow is likely to be laminar if  $R_e$  is less than a critical amount of about a million (Ref-3) and turbulent when  $R_e$  is larger than ten million with a transition region in between. These numbers are given only as examples; in specific cases the critical Reynolds number is usually determined experimentally. In what follows, it will be assumed that the flow is laminar.

The gases moving along a streamline through the line of symmetry in the gas cap ahead of the spherical forebody come to a standstill at the point of intersection of the symmetry axis and the outer boundary of the boundary layer. This point is known as the stagnation point. The streamline along which fluids flow from the freestream into the stagnation point is called the stagnation streamline. In the case of asymmetrical bodies, the stagnation point is usually located near the point where the tangent plane to the body surface is perpendicular to the free stream.

If the boundary layer thickness is small compared with the nose radius and compared with the shock standoff distance as well, then most of the fluids in the boundary layer will have entered it in the vicinity of the stagnation point. (See, e.g. Ref-4). These are gases that passed through the normal shock region and have been decelerated from hypersonic to subsonic speeds and finally slowed down practically to a standstill in the stagnation region. In this manner, the entire kinetic energy of translation of the free stream has been converted into thermal energy at the stagnation point. For an Apollo type reentry speed of 36,000 ft/sec this amounts to about 18 electron volts per air-molecule and is enough energy to dissociate most of the air molecules and to produce some ionization. The stagnation temperature reached is about 10,000°K to 11,000°K under these conditions.

In Figure 2 (From Ref-5) we give a plot of the relative contributions of dissociation and ionization energy to the gas enthalpy as a function of flight speed and stagnation pressure of 1 atmosphere.

## FORMULATION OF THE PROBLEM

In what follows, it will be assumed that the wall of the craft is cold\* relative to the gases flowing around it and that it is catalytic to chemical recombination. The former assumption implies that heat will flow from the gases to the body (instead of the other way around) and the latter implies that all dissociated atoms and ions that reach the cold wall will recombine there into molecules giving up their chemical energy to the wall.

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\*For a discussion of the fundamental principles involved, see Ref-6, for example.

The heat flux from the fluid to the wall consists in the following two distinct physical processes: linear thermal conduction proportional to the temperature gradient and chemical energy carried by diffusion proportional to the chemical concentration gradient. The expression for the heating rate  $\dot{q}$  (power per unit area) at the wall is then (Ref-7)

$$\dot{q} = - \left[ K \frac{\partial T}{\partial y} + \rho D \sum h_i \frac{\partial c_i}{\partial y} \right] \quad (1)$$

where  $K$  is the coefficient of thermal conductivity,  $T$  is the temperature,  $y$  is the coordinate perpendicular to the wall,  $\rho$  is the density,  $D$  is the binary diffusion coefficient,  $C_i$  is the chemical concentration (by weight) of species  $i$ , and  $h_i$  is the enthalpy of species  $i$ . The latter is defined as

$$h_i = \int_0^T C_{p_i} dT + h_i^\circ \quad (2)$$

where  $C_{p_i}$  is the specific heat at constant pressure of species  $i$  and  $h_i^\circ$  is the heat of formation of species  $i$ .

With the exception of the variable  $y$ , every one of the symbols on the right-hand side of Eq-1 depends on the detailed chemistry and thermodynamics of the gases in question which, in turn, depend on details defined by non-linear flow-field equations. The result is a system of simultaneous non-linear partial differential equations which are sufficiently complicated that any satisfactory solution has to be extracted numerically.

In order to gain a qualitative understanding of Eq-1, we shall reproduce an argument due to Lees (Ref-7).

One can rewrite (Ref-7) Equation-1 in the following approximate form\*

$$\dot{q}_w \approx \dot{Q}_w \left[ (1 - A) + (Le)_w A \right] \quad (3)$$

where the subscript  $w$  denotes conditions at the wall,  $\dot{Q}$  is an approximate expression for the equilibrium heating rate at the wall (i. e. the limiting value when the chemical reaction rates are infinitely fast).  $A$  is the ratio of the dissociation (and ionization) energy to the stagnation enthalpy and is of the order of unity for high enough flight speeds (at a flight speed of about 25,000 ft/sec and stagnation pressure of 1 atm, one obtains, from Fig-2,  $A \approx .7$ ). The Lewis number,  $Le$ , is the ratio of the diffusive to the conductive heat transport coefficient,

$$Le = \frac{\rho D \bar{C}_p}{K} \quad (4)$$

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\*See Appendix-A, for a derivation.

where  $\bar{C}_p$  is the specific heat at constant pressure of the gas (averaged over its various components).  $Le$  is of the order of unity and ranges (Ref-8) from about 1.4 at low temperatures to about .6 at 8,000°K; near the cold wall,  $Le$  will have a value close to 1.4. The last term on the right hand side of Eq-3 is the contribution to the heat flux by diffusion and the other term is the contribution by thermal conduction. It is clear that as the flight speed increases and  $A$  approaches unity, the relative importance of conduction diminishes and heat is mainly transported by diffusion. Also, if  $Le = 1$ , the terms involving  $A$  cancel, and one has the result that the heat flux is independent of the particular mode of energy transfer.

Thus, we may expect no great difference between the heating rates for equilibrium of frozen\* flows. This is verified by more rigorous calculations (Ref 5, 9) where it is shown that the frozen flow heating rates are higher than in the case of equilibrium flows but are of the same order of magnitude.

In the case of equilibrium flow, the dissociated molecules recombine some distance away from the wall, where the thermodynamic conditions become favorable for recombination. This produces a layer of molecules which has a tendency to insulate the wall from the hot boundary layer. In the case of frozen flow, however, this layer of recombined molecules is less effective an insulator since the atoms or ions will penetrate more deeply (because of their longer life time) and then recombine (catalytic wall) giving up their chemical energy much closer to the wall than in the equilibrium case. Fig-3 is a plot of the species concentration vs. the similarity variable as calculated by Fay and Kemp (Ref-5).

As a simple criterion to determine whether one is dealing with frozen or equilibrium flow, one may consider the magnitude of the flow-time  $t'$  relative to the recombination time  $\tau$  of the dissociated (and ionized) air. For the flow time  $t'$  one may take (Ref-9) the length of time required for the fluid to spread from the stagnation point over a distance of about a body radius. For a sphere with a strong shock and Newtonian flow, one has (Ref-10),

$$t' \simeq \frac{R}{V_\infty}$$

where  $V_\infty$  is the free stream velocity and  $R$  is the body radius.  $\tau$  can be evaluated from the relation

$$\tau^{-1} = K (n_0)^2$$

where  $K$  is the recombination rate coefficient and  $n_0$  is the atom concentration at the stagnation point; for Nitrogen (Ref-11)

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\*A flow field is said to be chemically frozen if the chemical reaction rates are so slow compared with the rate of flow that the concentration can be regarded constant along a given streamline.

$$K = 2.3 \times 10^{21} T^{-3/2} \left( \frac{\text{cm}^3}{\text{mole}} \right)^2 \text{sec}^{-1}.$$

If  $t' \gg \tau$ , atoms recombine fast compared with the rate of flow around the body and we have equilibrium flow while if  $t' \ll \tau$ , the opposite is true, namely the flow is fast compared with the chemical reaction rates and we are dealing with frozen flow.

### STAGNATION POINT HEATING

A general solution for the coupled nonlinear partial differential equations that need to be solved in order to obtain expressions for the heating rate has not been given in the literature (numerically or otherwise). In certain special cases, however, valid simplifications exist and numerical solution becomes feasible. Such is the case in the stagnation region, where all the flow-field variables except the velocity have a maximum. Using standard methods of approximations applicable near such an extremal point, numerous authors (Ref-10) have discussed such stagnation point solutions for the heat transfer problem.

Because of the large number of variables involved, it is important that the calculated results have as wide a range of applicability as possible. In other words, one seeks a combination of variables which is invariant for as large a number of geometrical shapes and flight conditions as possible (i. e., a scaling factor) and yet be of such a form that with simple mathematical operations one can obtain from it the heating rates for special cases of interest. Such a scaling factor is the heat transfer parameter\*, defined as the Nusselt number divided by the square root of the Reynolds number,  $Nu/\sqrt{Re}$ , and is given by

$$\frac{Nu}{\sqrt{Re}} = \frac{\dot{q}_o}{(h_o - h_w) \sqrt{\frac{du}{dx}}} \left( \frac{P_r}{\sqrt{\rho\mu}} \right)^w \quad (5)$$

where the subscript o denotes conditions at the stagnation point,  $du/dx$  is the velocity gradient along the surface,  $P_r$  is the Prandl number  $P_r = \frac{C_p \mu}{K}$  and  $\mu$  is

the viscosity. The heat transfer parameter Eq-5 is not, strictly speaking, a constant, but is a fairly weak function of the flow field parameters.

Thusly, if the function  $Nu/\sqrt{Re}$  is known,  $\dot{q}_o$  can be computed by simply solving Eq-5 for  $\dot{q}_o$ ;

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\*See Ref-6, for example.

$$\dot{q}_o = \left( \frac{\sqrt{\rho\mu}}{Pr} \right)_w (h_o - h_w) \sqrt{(du/dx)_o} (Nu/\sqrt{Re})$$

For a given flight condition, the free stream velocity,  $V_\infty$ , the altitude  $H$ , the body shape (i. e. effective body radius  $R$ ) and the wall temperature  $T_w$  are defined. Knowing  $V_\infty$  and  $H$  one can look up  $P_o$  from Ref-12 and 13. Once  $P_o$  is found one can compute the stagnation pressure  $P_o$  using Lighthill's approximation Ref-14 for a strong shock,

$$P_o = (1 - P_\infty/p_o) P_\infty V_\infty^2$$

Assuming the pressure at the wall in the stagnation region equals the pressure at the stagnation point, we can calculate  $P_w$  from the ideal gas law.  $\mu_w$  and  $Pr_w$  can be obtained from Ref-15;  $h_o$  can be found from the relation

$$h_o = 1/2 V_\infty^2 + h_\infty.$$

$h_\infty$  and  $h_w$  are functions of temperature only and can be obtained from Ref 12 and 13;  $h_o$  is usually very much larger than either  $h_\infty$  or  $h_w$ . The stagnation point velocity gradient  $(du/dx)_o$  is reasonably approximated by the Newtonian formula

$$\left( \frac{du}{dx} \right)_o = \frac{1}{R} \sqrt{2 \frac{(P_o - P_\infty)}{\rho_o}}$$

where the free stream hydrostatic pressure  $P_\infty$  is given in Ref 16.

Figure 4 is a comparison of theory with experiment based on a plot by Rose and Stankevics (Ref-17). The explicit correlation formulae proposed by the various authors are given in the Appendix. It is seen that the various theories (Ref-5 and 18-21) give values for  $Nu/\sqrt{Re}$  that are within  $\pm 25\%$  with each other, up to velocities of 55,000 ft/sec except the one by Scala (Ref-21) which follows a different trend altogether at flight speeds in excess of about 25,000 ft/sec.

The major source for the discrepancy between the various theories is believed to be due to the particular transport properties used. There are serious gaps in experimental information of the high temperature properties of air and assumed or estimated properties (e. g. charge exchange collision crosssections for  $N-N^+$  and  $O-O^+$ ) have to be used.

The large disagreement between the calculations of Scala (Ref-21) with the general trend is believed (Ref-5) to be due to the small charge exchange cross sections he used which are one to two orders of magnitude smaller than the recent estimates of Yos (Ref-15). This gives rise to a very high

thermal conductivity\* which is one or two orders of magnitude higher than that calculated by Yos in the range from 10,000°K to 20,000°K with a corresponding effect on the heating rates.

In their calculations for the equilibrium flow of air, Pallone (Ref-19), Hoshizake (Ref-18), and Cohen (Ref-20) used the transport properties of Hansen (Ref-8); therefore these calculations should be equivalent and hence, the difference in their correlation formulae which may be as large as 30% is to be attributed to the different computational procedures they employed. Hansen computed the high temperature crosssections on the basis of polarizability and this procedure has recently been criticized by Fay (Ref-10) since the high temperature crosssections are mainly due to the short range repulsive forces and not to the attractive induced dipole forces. This latter effect is important only at temperatures below 1000°K.

In an effort to improve the existing state-of-the-art and to carefully investigate the frozen flow properties, Fay and Kemp (Ref-5) proposed a binary diffusion approximation in which the charge exchange crosssections are assumed to be very large compared with the kinetic crosssections. This is a realistic model in view of the fact that the charge exchange crosssections are believed to be about an order of magnitude larger than the kinetic crosssections. Figure-5 is a comparison of equilibrium thermal conductivities of Hansen (Ref-8), Fay and Kemp (Ref-5) and the recent calculations of Yos (Ref-15). The latter authors are seen to be in good agreement with each other.

Returning to Figure-4, we see there is good agreement between the heat transfer parameters calculated by Fay and Kemp and experiment. It is interesting to note the close agreement between the equilibrium calculations of Fay and Kemp with the  $N_2$  calculations of Pallone and Van Tassell where they used Yos' transport properties. The experimental points, however, scatter considerably and it is not possible to prefer one theory to the other on the basis of comparison with experiment, since all the theories plotted (except Scala's calculation) are within the range of the scatter of the data and this scatter has not as yet been explained. Errors in the reduction of the raw experimental data may be due to uncertainties in the tunnel conditions as well as to incomplete knowledge of the radiative contribution (Ref. 22-23) to the heat load measured by the instruments. The validity of Warren's (Ref-24) data has been openly questioned by Rose and Stankevics (Ref-17) on fundamental grounds and these authors regard Warren's data as anomalous. It is, however, clear that the correlation formula of Fay and Kemp give the more conservative estimates; Hoshizaki's correlation formula (used by NAA for Apollo design purposes) many underestimate the heating rate by over 30% in the lower velocity range.

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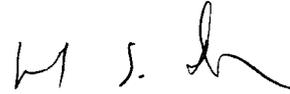
\*This can be seen qualitatively, because if the collision crosssection corresponding to a certain type of collision is small, there will be few collisions (due to this process) that diminish the transport of heat by diffusion. Hence, the conduction of heat (due to diffusion) is higher than would be the case for large collision crosssections.

## CONCLUSION

The various theoretical correlation formulas for the heat transfer parameter agree with each other to within plus or minus 25% except the one by Scala. This discrepancy is believed to be due to the very small charge exchange cross-sections used in Scala's calculations. Use of the more recent estimations of the gas transport properties by Yos and by Fay and Kemp lead to correlation formulas predicting higher heating rates than do calculations based on earlier estimates of transport properties by Hansen.

The calculations using the more recent transport properties are applicable to an atmosphere of pure Nitrogen. It is believed, however, that in view of the uncertainties inherent in the approximations employed, these calculations should be equally valid for air.

The various correlation formulas (except the one by Scala) agree with experiment. Since, however, the scatter in the experimental points approximately overlaps with the spread in the theoretical values, it is not possible to select any one theory as the best one on the basis of the experimental information alone. It may, however, be pointed out that the calculations of Fay and Kemp, as well as those of Pallone for Nitrogen make use of the most recent estimates of the transport coefficients and give conservative estimates for the heat transfer parameters.



J. S. Dohnanyi

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## Attached:

Figures 1-5  
Appendixes A and B

## cc:

J. A. Hornbeck  
M. W. Hunter II  
H. S. London  
C. A. Lovell  
C. R. Moster  
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APPENDIX A

In this Appendix, we shall reproduce Lees' derivation (Ref-7) of Eq-3. One may start with Eq-1:

$$(\dot{q})_w = - \left( K \frac{\partial T}{\partial y} + D \sum h_i \frac{\partial C_i}{\partial y} \right)_w \quad (1)$$

and rewrite it in an approximate form for the case of complete thermodynamic equilibrium. In this case, the atom concentration vanishes at the wall, i. e.,  $(\partial C_i / \partial y)_w = 0$ . Since  $dh = \bar{C}_p dT + \sum h_i dC_i$  where  $\bar{C}_p = \sum C_i C_{pi}$ , the first term on the right hand side of Eq-1 can be rewritten approximately as

$$- \left( K \frac{\partial T}{\partial y} \right)_w = - \left( \frac{K}{\bar{C}_p} \frac{\partial h}{\partial y} \right)_w \approx - \left( \frac{K}{\bar{C}_p} \right)_w \frac{h_o - h_w}{\delta} \quad (A-1)$$

where  $\delta$  is the thickness of the boundary layer.

Substituting (A-1) into Eq-1 and remembering that  $(\partial C_i / \partial y)_w \approx 0$ , we have

$$\dot{q}_{w, \text{ equilibrium}} = \dot{q}_w \approx \left( \frac{K}{\bar{C}_p} \right)_w \frac{h_o - h_w}{\delta} \quad (A-2)$$

In the opposite limiting case, where the recombination rates are low compared with the diffusion rates, one has for the atoms  $(\partial C_i / \partial y)_w \approx C_{io} / \delta$ , on the assumption that the atom concentration vanishes at the wall (catalytic wall) and for the molecules,  $(\partial C_i / \partial y)_w \approx (C_{io} - C_{iw})$ .

Substituting these relations together with A-1 into Eq-1, one has

$$\dot{q}_w \approx \left( \frac{K}{\bar{C}_p} \right)_w \frac{h_o - h_w}{\delta} \left[ \left\{ 1 - \frac{\sum h_{iw} (C_{io} - C_{iw})}{h_o - h_w} \right\} + \right. \\ \left. + Le_w \frac{\sum h_{iw} (C_{io} - C_{iw})}{h_o - h_w} \right] \quad (A-3)$$

Consider now the numerator of the terms involving the chemical concentrations.

$$\sum_i h_{iw} (C_{io} - C_{iw}) = h_{m,w} (C_{m,o} - C_{m,w}) + h_{a,w} C_{a,o} \quad (A-4)$$

where the subscripts m and a stand for molecules and atoms, respectively. Since we have a catalytic wall,  $C_{m,w} = 1$  and assuming diatomic molecules  $1 - C_m = C_a$ . Hence, the right-hand side of (A-4) becomes

$$-h_{m,w} C_{a,o} + h_{a,w} C_{a,o} \quad (A-5)$$

Since, however, the enthalpy of the molecules is equal to the sum of the energy of the atoms and the (negative) dissociation energy, (A-5) reduces to

$$h^{\circ} C_{ao} \quad (A-6)$$

where  $h^{\circ}$  is the energy of formation for the atoms per unit mass. Consequently, the fractions involving the concentrations appearing on the right hand side of (A-3) can be rewritten as

$$\frac{\sum_i h_{iw} (C_{io} - C_{iw})}{h_o - h_w} \approx \frac{h^{\circ} C_{ao}}{h_o - h_w} \approx \frac{h^{\circ} C_{ao}}{1/2 V_{\infty}^2} \quad (A-7)$$

where the last equality follows because  $h_o \approx 1/2 V_{\infty}^2 \gg h_w$ . (A-7) is the ratio of the dissociation energy density to the total stagnation point energy density denoted by A in the text. Using A-2 one can now rewrite (A-3) to give Eq-3.

$$q_w \approx Q_w [(1-A) + Le_w A] \quad (3)$$

and the derivation is complete.

The foregoing discussion does not include ionization. A simple extension of the treatment shows, however, that if ionization is included, its result will be a redefinition of the quantity A in Eq-3.

$$A \rightarrow A^1 = \frac{h^{\circ} C_{ao} + (h^{\circ} + h^I) C_{Io}}{1/2 V_{\infty}^2}$$

where  $h^I$  and  $C_{Io}$  are the ionization energy and ion concentration, respectively.

APPENDIX B

Here we list the correlation formulas plotted in Fig. -4. These formulas are empirical fits through points that have been obtained numerically from the theoretical models employed by the respective authors.

Hoshizaki's (Ref-18) correlation formula is independent of altitude and is applicable for speeds between 6000 ft/sec and 50,000 ft/sec and is given by:

$$\frac{N_u}{\sqrt{R_e}} = .478 \left( \frac{T_w}{500^\circ K} \right)^{.2} \left( \frac{V_\infty}{10^4 \text{ ft/sec}} \right)^{-.31}$$

Cohen (Ref-10) obtained a relation of the form

$$\frac{N_u}{\sqrt{R_e}} = .767 \left( \frac{\rho_s \mu_s}{\rho_w \mu_w} \right)^{.43} \left[ 1 + .3 \left( \frac{V_\infty}{30,000} - 1 \right) \right]^2$$

where the last factor is to be replaced by unity for  $V_\infty < 30,000$  ft/sec.

Pallone and Van Tassell (Ref-19) obtained the following correlation for air

$$\frac{N_u}{\sqrt{R_e}} = .90 (\text{Pr}_w)^{1/4} \left( \frac{\rho_o \mu_o}{\rho_w \mu_w} \right)^{.43} \left( \frac{V_\infty}{32,500} \right)$$

and for Nitrogen

$$\frac{N_u}{\sqrt{R_e}} = .915 (\text{Pr}_w)^{1/4} \left( \frac{\rho_o \mu_o}{\rho_w \mu_w} \right)^{.43} \left( \frac{V_\infty}{37,500} \right)^{1.15(P)^{-.04}}$$

where P is the stagnation pressure in atmospheres. They used Hansen's transport properties in their calculations for air and Yos's transport properties for Nitrogen.

Fay and Kemp (Ref-5) obtain for the equilibrium flow of Nitrogen

$$\frac{N_u}{\sqrt{R_e}} = .47 \quad \left( \frac{V_\infty}{24} \right)^{-N} \quad \begin{array}{l} 10 < V_\infty < 24 \\ 24 < V_\infty < 60 \end{array}$$

where  $N = .38 - .11 (\log P_s) - .02 (\log P_s)^2$

where  $V_\infty$  is in  $10^3$  ft/sec and  $\log P_s$  is the logarithm to the base ten of the stagnation pressure in atmospheres. For frozen flow, Fay and Kemp obtain

$$\begin{array}{ll} \frac{N_u}{\sqrt{Re}} = .47 & 10 < V_\infty < 24 \\ \frac{N_u}{\sqrt{Re}} = .47 (V_\infty/24)^{-.83} & 24 < V_\infty < 34 \\ = .35 & 34 < V_\infty < 60 \end{array}$$

NOMENCLATURE

A	ratio of dissociation and ionization energies to the stagnation enthalpy of the free stream.
$C_i$	concentration (by weight) of species i
$C_p$	specific heat at constant pressure
D	binary diffusion coefficient
h	enthalpy per unit mass
H	altitude
K	coefficient of thermal conductivity
l	characteristic body length
$L_e$	Lewis number = $D \bar{C}_p / K$
n	atom molar concentration
$N_u / R_e$	heat transfer parameter
P	pressure
$P_r$	Prandtl number = $\frac{C_p \mu}{K}$
$\dot{q}, \dot{Q}$	heat flux
R	body radius
$R_e$	Reynolds number = $\frac{\rho V l}{\mu}$
T	temperature
$\mu$	velocity in the boundary layer
V	velocity
x	coordinate along the body surface
y	coordinate along the normal to the body surface
$\eta$	similarity coordinate normal to body surface
$\rho$	density
$\mu$	viscosity

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viscosity

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- i denotes species i
- o denoted conditions at the stagnation point
- w denotes conditions at the body wall
- $\infty$  denotes conditions in the free stream

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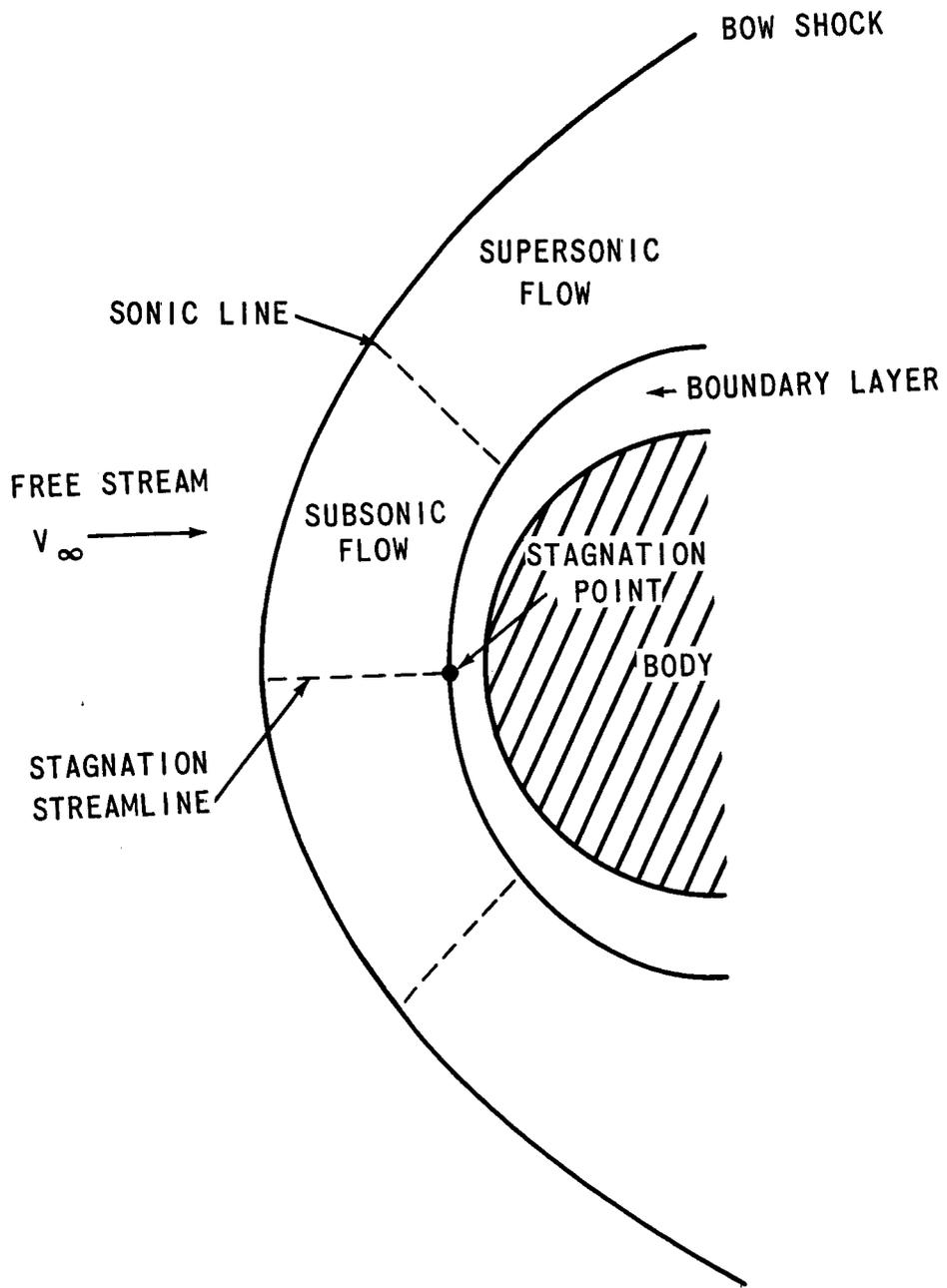


FIGURE 1

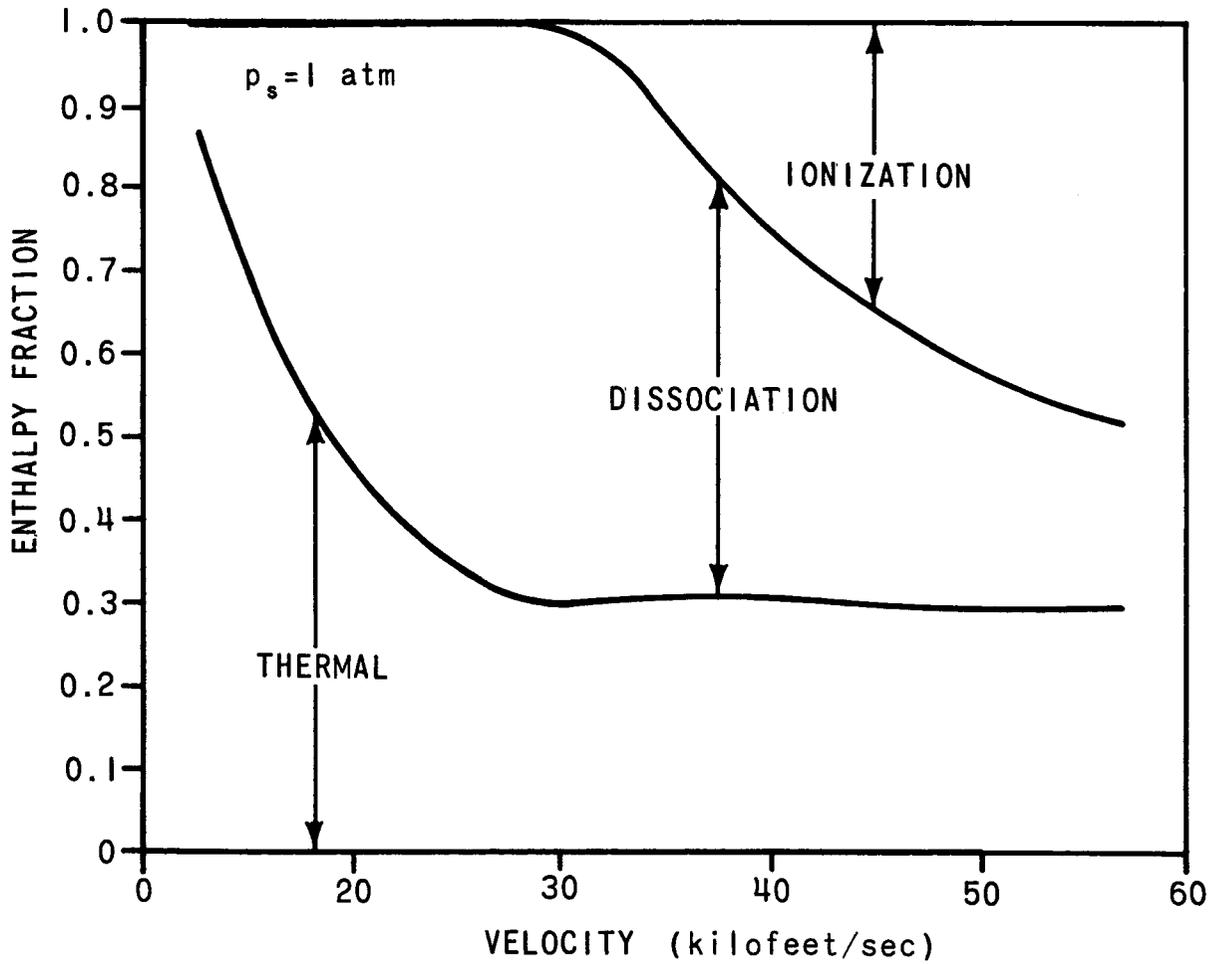


FIGURE 2 FRACTIONAL DISTRIBUTION OF EQUILIBRIUM STAGNATION ENTHALPY AMONG THERMAL, DISSOCIATIVE, AND IONIC MODES FOR NITROGEN AT A PRESSURE OF 1 atm.

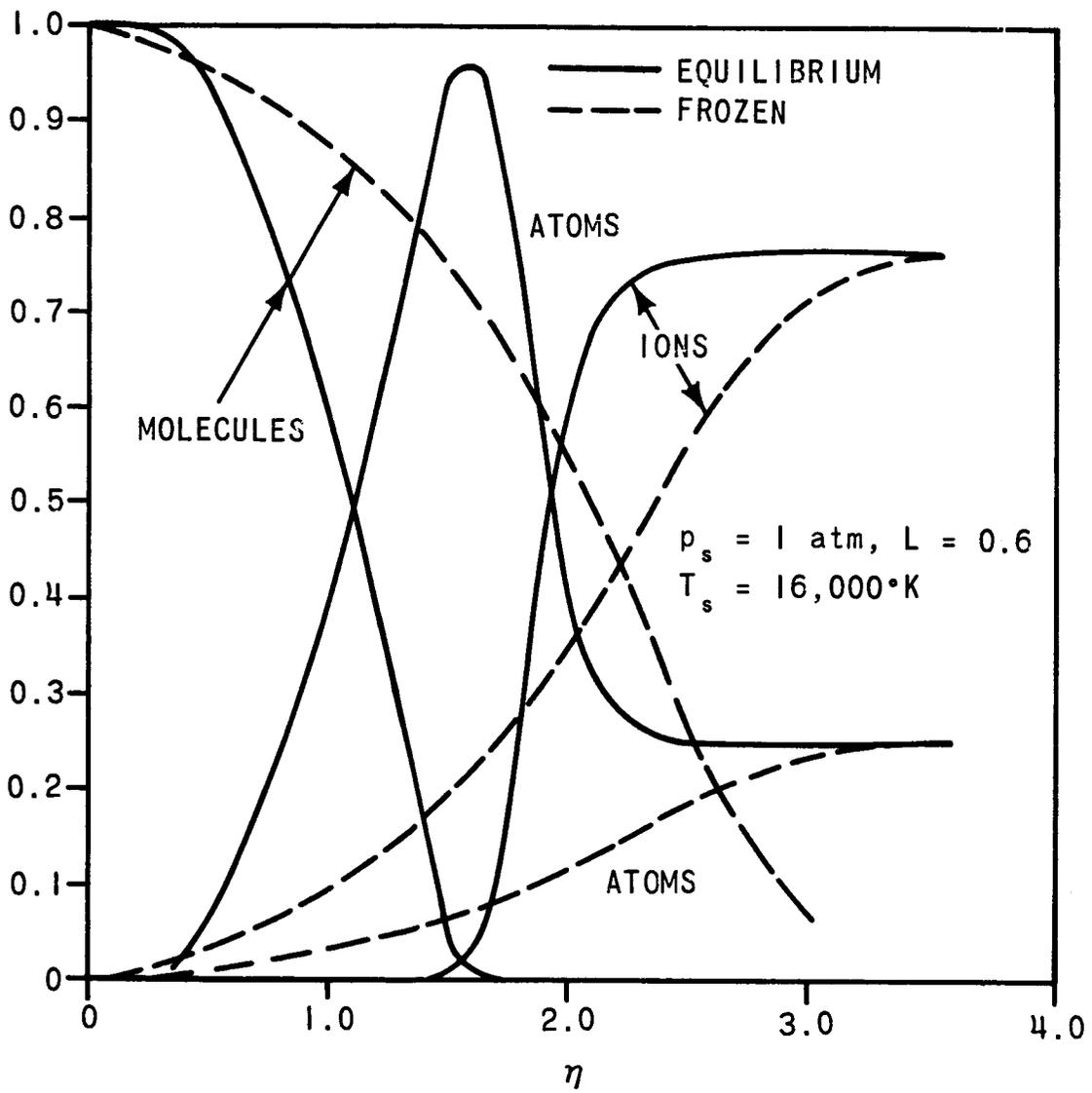


FIGURE 3 SPECIES MASS FRACTION DISTRIBUTIONS FOR  $p_s = 1 \text{ atm}$ ,  $L = 0.6$ , AND  $T_s = 16,000^\circ\text{K}$

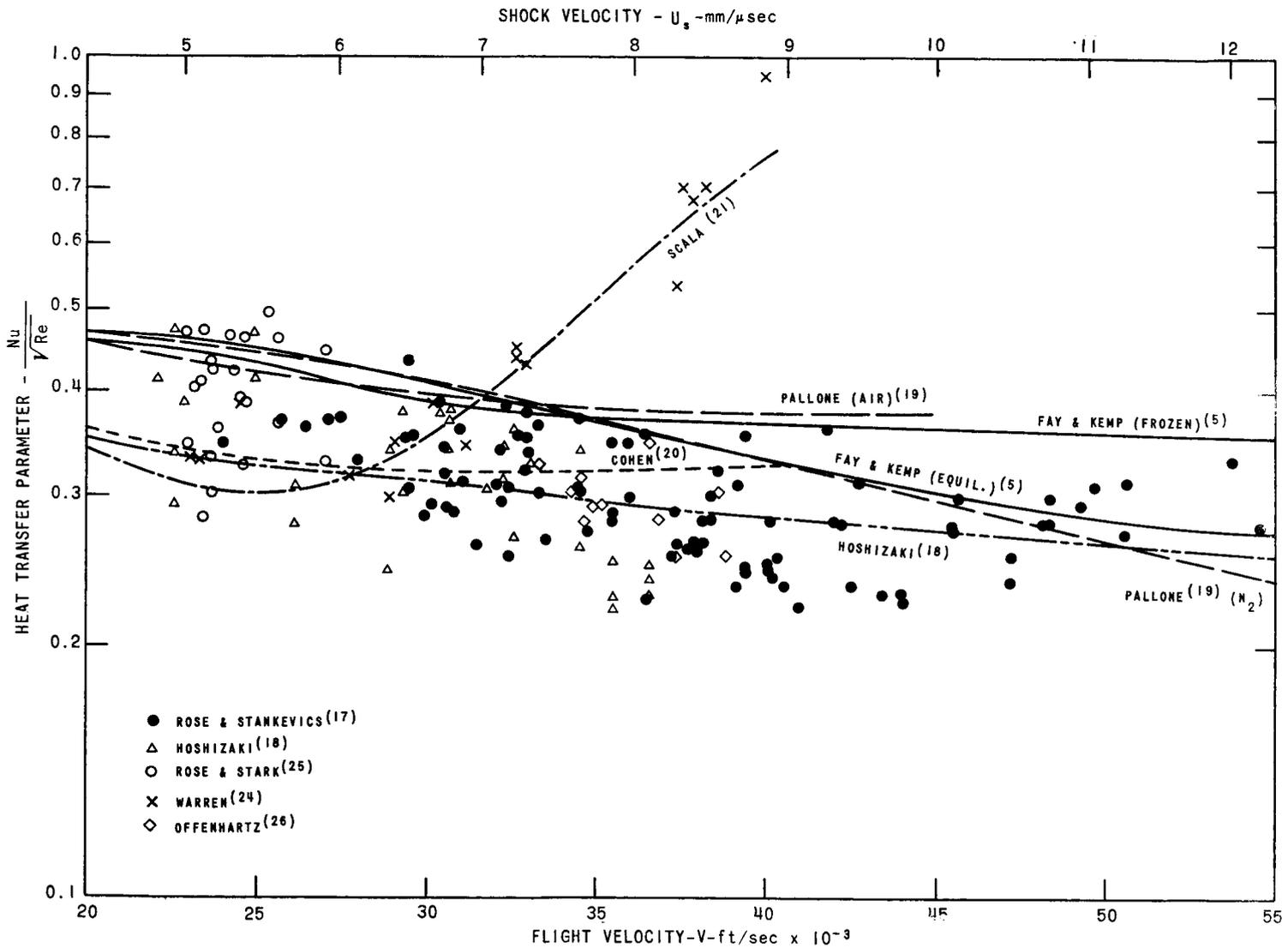


FIGURE 4 SUMMARY PLOT OF VARIOUS EXPERIMENTAL DATA AND THEORIES FOR STAGNATION POINT HEAT TRANSFER IN PARTIALLY IONIZED AIR AND NITROGEN. PALLONE (19), HOSHIZAKI (18) AND COHEN (20) THEORIES ARE SHOWN APPLICABLE TO A STAGNATION PRESSURE OF 1 atm. FAY AND KEMP (5) WAS CALCULATED FOR THE EXPERIMENTAL CONDITIONS, i.e.,  $p_s = 0.25$  mm OF Hg. SCALA (21) WAS CALCULATED FOR A FLIGHT ALTITUDE OF 240,000 ft.

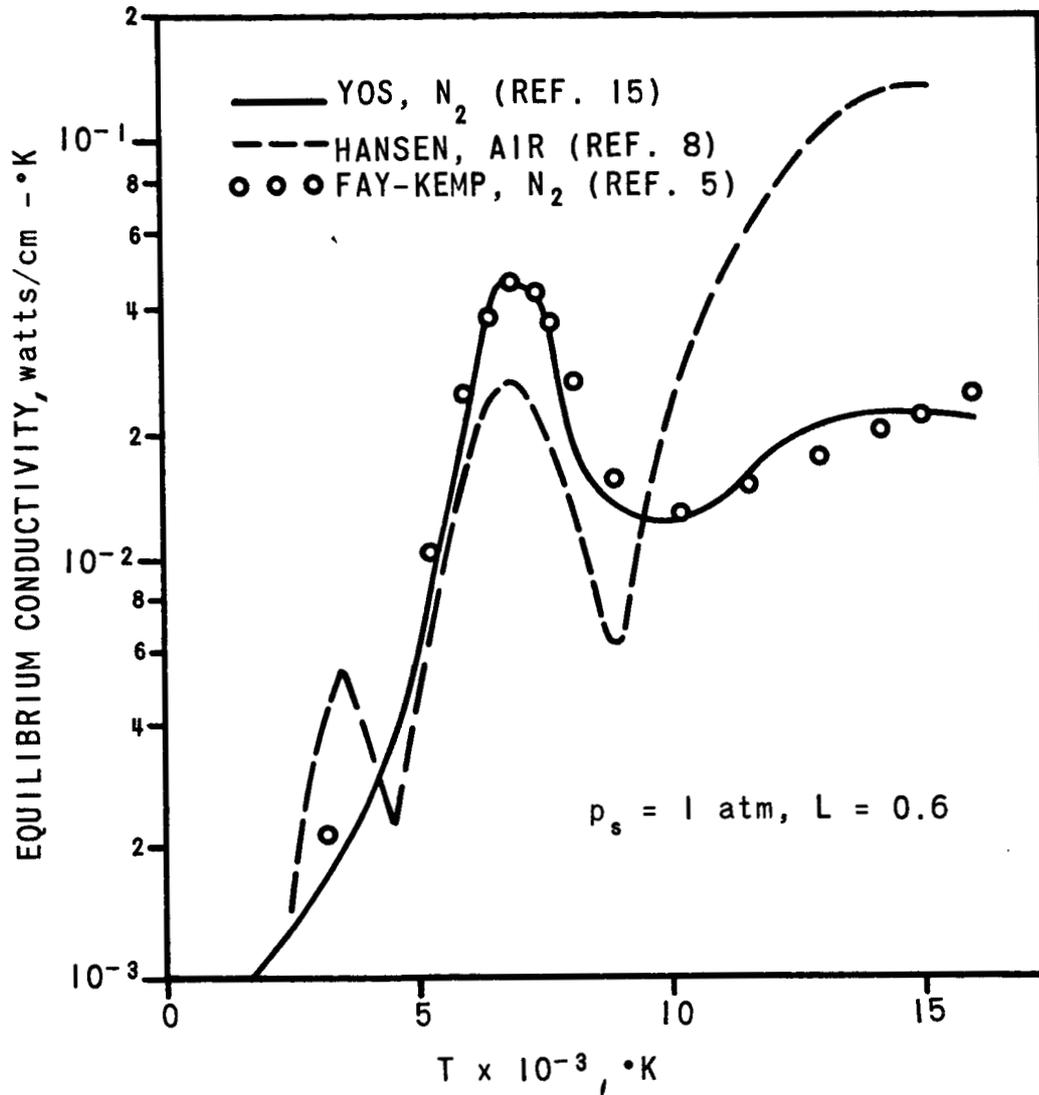


FIGURE 5 COMPARISON OF EQUILIBRIUM THERMAL CONDUCTIVITY USED IN SEVERAL HEAT-TRANSFER CALCULATIONS.